A State Space Method for Solving Differential Algebraic Equations with Nonholonomic Constraints

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EXTENDED ABSTRACT

This paper focuses on the state space method based on singular value decomposition to solve the dynamic differential algebraic equations of multibody systems with holonomic and nonholonomic constraints. The dynamic equations of multibody systems as the typical constrained systems can usually be expressed in the form of ordinary differential equations (ODEs) or differential algebraic equations (DAEs). At present, the numerical solution technology of ODEs has been very mature, but there are still some difficulties in the numerical calculation of differential algebraic equations [1], and the numerical methods of differential algebraic equations with nonholonomic constraints is even less. Due to its non-integrability, the mathematical model of nonholonomic constraints is a group of implicit and underdetermined ordinary differential equation. At present, little attention has been paid to solve the DAEs with nonholonomic constraints. However, nonholonomic constraints generally exist in the control of mechanical systems. Therefore, the numerical methods for solving the DAEs of multibody system dynamics with nonholonomic constraints are not only very important in theory, but also have high application value.

The state space method is a numerical method to solve the dynamic DAEs of multibody systems. When the system is serious stiff, the implicit method needs to be used for integrating the dynamic DAEs of multibody systems. In the process of solving, there are the cycle of solving implicit integration methods and the cycle of solving nonlinear algebraic constraint equations, that is, there are two cycles[2]. In this paper, the cycle of solving the constraint equations is embedded in the implicit integration process, and a state space method based on singular value decomposition is proposed to solve the DAEs with only holonomic constraints and with nonholonomic constraints, respectively.

No matter what type of multibody system dynamics DAEs, their acceleration constraint equations are linear equations, so the dynamic DAEs of multibody systems in the form of index 1 have a unified form:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}}+\mathbf{C}_{\mathbf{q}}^{T}\boldsymbol{\lambda}=\mathbf{Q} \\ \mathbf{C}_{\mathbf{q}}\ddot{\mathbf{q}}=\mathbf{Q}_{\mathbf{a}} \end{cases} \tag{1}$$

where $\mathbf{q}$ is the generalized coordinates, $\mathbf{M}$ is the mass matrix, $\mathbf{C}_{\mathbf{q}}$ is the constrained Jacobian matrix, $\boldsymbol{\lambda}$ is the Lagrange multiplier, and $\mathbf{Q}$ is the generalized force. $\mathbf{Q}_{\mathbf{a}}$=-[$\left(\mathbf{C}_{\mathbf{q}}\dot{\mathbf{q}}\right)_{\mathbf{q}}\dot{\mathbf{q}}+2\mathbf{C}_{\mathbf{qt}}\dot{\mathbf{q}}+\mathbf{C}_{\mathbf{tt}}$], $\mathbf{C}_{\mathbf{qt}}=\partial\mathbf{C}_{\mathbf{q}}/\partial t$, $\mathbf{C}_{\mathbf{tt}}=\partial^2\mathbf{C}/\partial t^2$, $\mathbf{C}$ is the constraint matrix.

The acceleration can be obtained from the differential algebraic equations in the form of index 1:

$$\ddot{\mathbf{q}}=\mathbf{M}^{-1}\mathbf{Q}-\mathbf{M}^{-1}\mathbf{C}_{\mathbf{q}}^{T}\boldsymbol{\lambda}=\mathbf{M}^{-1}\mathbf{Q}-\mathbf{M}^{-1}\mathbf{C}_{\mathbf{q}}^{T}(\mathbf{C}_{\mathbf{q}}\mathbf{M}^{-1}\mathbf{C}_{\mathbf{q}}^{T})^{-1}(\mathbf{C}_{\mathbf{q}}\mathbf{M}^{-1}\mathbf{Q}-\mathbf{Q}_{\mathbf{a}}) \tag{2}$$

In dynamic problems, the first order nonholonomic constraints are velocity constraints in nature. The classical state space method cannot be used to solve DAEs with nonholonomic constraints because the number of ODEs with respect to the simplest coordinates in the state space is less than the number of independent coordinates. So we use the integral method to supplement the algebraic relation with the number of nonholonomic constraints and construct a new solution scheme for DAEs with nonholonomic constraints based on the classical state space method. Firstly, the DAEs is transformed into ODEs of form Eq.(2) by index reduction processing, and then, Eq.(2) was directly integrated and the coordinates were separated by singular value decomposition(SVD) [3] to obtain independent velocities and positions. In this case, the time integral of the dependent variable can provide good initial values for the iteration of the velocity constraint equations and position constraint equations. Then, a state space is defined by the holonomic constraint, and the dependent position coordinates are obtained by solving the position constraint equations. Finally, another state space is defined by all the constraint equations, and the velocity constraint equations are solved to obtain the dependence velocity coordinates.

If only holonomic constraints are included, the processing of nonholonomic constraints can be skipped, and Eq.(2) can be directly integrated to obtain independent coordinates. Then the position constraint equations and velocity constraint equations can be solved to obtain dependent coordinates. When solving these two kinds of constraint equations, Boolean matrix can be introduced to avoid the rearrangement of the constraint equations[4]. Note that when integrating all variables in Eq.(2), it is not necessary to rearrange the ordinary differential part of the DAEs. So all equations in this state space method do not need to be rearranged. The flow of the algorithm in one time step is shown in Fig. 1.

In this paper, the proposed method is used to simulate the dynamics of a flexible simple pendulum with planar motion as shown in Figure 2, and the simulation results are compared with those of the explicit Runge-Kutta method.

After 1.2s simulation, the end lateral displacement and lateral deformation of the flexible beam are shown in the Fig.3 and Fig.4.

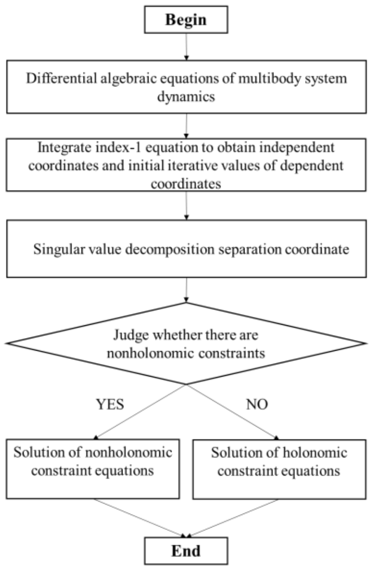

Fig. 1 Flow chart of the proposed method

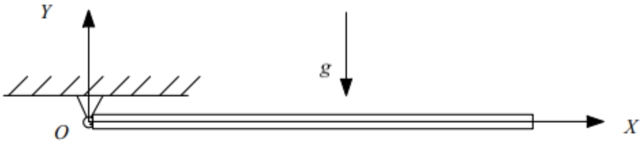

Fig. 2 flexible simple pendulum in plane motion

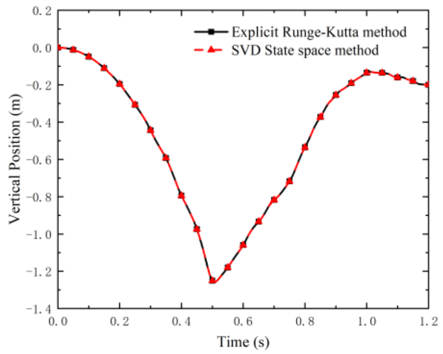

Fig. 3 Lateral displacement of the end point of the flexible beam

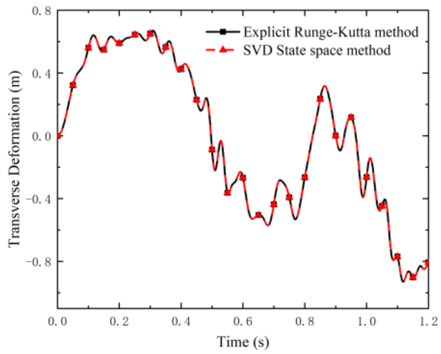

Fig. 4 Transverse deformation of the end point of flexible beam

The state space method based on singular value decomposition constructed in this paper can effectively solve the DAEs of flexible multibody system dynamics with holonomic constraints and nonholonomic constraints in theory, and has ideal results in accuracy, stability and computational efficiency.

Acknowledgments

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